

Implications of final state interactions in B -decays

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Abstract. I give a brief review of final state interactions (FSI) in heavy meson decays, paying particular attention to B -meson physics. Available theoretical methods for dealing with the effects of FSI are discussed.

1. Introduction

Strong interaction phases play an important role in the decays of heavy mesons. They produce visible effects in many nonleptonic decays and could be important for the proper interpretation of effects of underlying fundamental physics. For example, strong phases between isospin amplitudes, $\delta_{1/2}$ and $\delta_{3/2}$,

$$\begin{aligned} \mathcal{A}(\bar{B}^0 \rightarrow D^+ \pi^-) &= \sqrt{\frac{1}{3}} |A_{3/2}| e^{i\delta_{3/2}} + \sqrt{\frac{2}{3}} |A_{1/2}| e^{i\delta_{1/2}}, \\ \mathcal{A}(\bar{B}^0 \rightarrow D^0 \pi^0) &= \sqrt{\frac{2}{3}} |A_{3/2}| e^{i\delta_{3/2}} - \sqrt{\frac{1}{3}} |A_{1/2}| e^{i\delta_{1/2}}, \\ \mathcal{A}(B^- \rightarrow D^0 \pi^-) &= \sqrt{3} |A_{3/2}| e^{i\delta_{3/2}}, \end{aligned} \quad (1)$$

affect branching ratios of individual decays, as well as ratios of rates of isospin-related transitions. More importantly, they complicate interpretations of CP-violating phases from $\Delta b = 1$ transitions observed in the so-called direct CP-violating asymmetries. Provided that B-decay amplitude depends on at least two amplitudes with different weak and strong phases (for example, tree $A_1 = \mathcal{T}$ and penguin $A_2 = \mathcal{P}$ amplitudes),

$$\begin{aligned} \mathcal{A}(B \rightarrow f) &= A_1 e^{i\phi_1} e^{i\delta_1} + A_2 e^{i\phi_2} e^{i\delta_2} \\ \mathcal{A}(\bar{B} \rightarrow \bar{f}) &= A_1 e^{-i\phi_1} e^{i\delta_1} + A_2 e^{-i\phi_2} e^{i\delta_2}, \end{aligned} \quad (2)$$

a CP-violating asymmetry can be formed,

$$A_{CP} = \frac{\Gamma(B \rightarrow f) - \Gamma(\bar{B} \rightarrow \bar{f})}{\Gamma(B \rightarrow f) + \Gamma(\bar{B} \rightarrow \bar{f})} = \frac{2A_1 A_2 \sin(\phi_1 - \phi_2) \sin(\delta_1 - \delta_2)}{A_1^2 + A_2^2 + 2A_1 A_2 \cos(\phi_1 - \phi_2) \cos(\delta_1 - \delta_2)}, \quad (3)$$

which clearly depends on both, CP-conserving $\Delta\phi = \phi_1 - \phi_2$ and CP-violating phase $\Delta\delta = \delta_1 - \delta_2$ differences. CP-conserving phase difference is associated with strong interactions. There are, of course, many more examples. It is therefore important to have a way of computing those phases, which in general would depend on the meson system under consideration.

The difference of the physical picture at the energy scales relevant to K , D and B decays calls for a specific descriptions for each class of decays. For instance, the relevant energy scale in K decays is $m_K \ll 1$ GeV. With such a low energy release only a few final state channels are available. This significantly simplifies the theoretical understanding of FSI in kaon decays. In addition, chiral symmetry can also be employed to assist the theoretical description of FSI in K decays. In D decays, the relevant scale is $m_D \sim 1$ GeV. This region is populated by the light quark resonances, so one might expect their significant influence on the decay rates and CP -violating asymmetries. No model-independent description of FSI is available, but it is hinted at experimentally that the number of available channels is still limited, allowing for a modeling of the relevant QCD dynamics. Finally, in B decays, where the relevant energy scale $m_B \gg 1$ GeV is well above the resonance region, the heavy quark limit might prove useful.

Final state interactions in $A \rightarrow f$ arise as a consequence of the unitarity of the \mathcal{S} -matrix, $\mathcal{S}^\dagger \mathcal{S} = 1$, and involve the rescattering of physical particles in the final state. The \mathcal{T} -matrix, $\mathcal{T} = i(1 - \mathcal{S})$, obeys the optical theorem:

$$\mathcal{D}isc \mathcal{T}_{A \rightarrow f} \equiv \frac{1}{2i} \left[\langle f | \mathcal{T} | A \rangle - \langle f | \mathcal{T}^\dagger | A \rangle \right] = \frac{1}{2} \sum_i \langle f | \mathcal{T}^\dagger | i \rangle \langle i | \mathcal{T} | A \rangle , \quad (4)$$

where $\mathcal{D}isc$ denotes the discontinuity across physical cut. Using CPT in the form $\langle \bar{f} | \mathcal{T} | \bar{A} \rangle^* = \langle \bar{A} | \mathcal{T}^\dagger | \bar{f} \rangle = \langle f | \mathcal{T}^\dagger | A \rangle$, this can be transformed into

$$\langle \bar{f} | \mathcal{T} | \bar{A} \rangle^* = \sum_i \langle f | \mathcal{S}^\dagger | i \rangle \langle i | \mathcal{T} | A \rangle . \quad (5)$$

Here, the states $|i\rangle$ represent all possible final states (including $|f\rangle$) which can be reached from the state $|A\rangle$ by the weak transition matrix \mathcal{T} . The right hand side of Eq. (5) can then be viewed as a weak decay of $|A\rangle$ into $|i\rangle$ followed by a strong rescattering of $|i\rangle$ into $|f\rangle$. Thus, we identify $\langle f | \mathcal{S}^\dagger | i \rangle$ as a FSI rescattering of particles. Notice that if $|i\rangle$ is an eigenstate of \mathcal{S} with a phase $e^{2i\delta}$, we have

$$\langle \bar{i} | \mathcal{T} | \bar{A} \rangle^* = e^{-2i\delta_i} \langle i | \mathcal{T} | A \rangle . \quad (6)$$

which implies equal rates for the charge conjugated decays¹. Also

$$\langle \bar{i} | \mathcal{T} | \bar{B} \rangle = e^{i\delta} T_i \langle i | \mathcal{T} | A \rangle = e^{i\delta} T_i^* \quad (7)$$

The matrix elements T_i are assumed to be the “bare” decay amplitudes and have no rescattering phases. This implies that these transition matrix elements between charge conjugated states are just the complex conjugated ones of each other. Eq. (7) is known as Watson’s theorem [1]. Note that the problem of unambiguous separation of “true bare” amplitudes from the “true FSI” ones (known as Omnés problem) was solved only for a limited number of cases.

While the above discussion gives the most general way of determining strong phases, especially if an S-matrix is easily diagonalized, it might not be the most practical way of dealing with strong phases in B -decays due to the large number of available channels.

2. Decays of heavy flavors

Theoretical analysis of decays of heavy-flavored mesons, in particular B -mesons, simplifies in the limit $m_b \rightarrow \infty$. In this limit, $q\bar{q}$ -pair produced in the weak decay of a b -quark, emerges as a small color dipole. This is a reasonable assumption, as the length scale of $q\bar{q}$ production is set by the inverse heavy quark mass, while soft QCD interactions are governed by the length

¹ This fact will be important in the studies of CP -violating asymmetries as no CP asymmetry is generated in this case.

scale associated with $1/\Lambda_{QCD}$, and so their effects will be suppressed by powers of Λ_{QCD}/m_b . Then, if $B \rightarrow M_1 M_2$ decay amplitude is dominated by this two-body-like configuration with small invariant mass, a factorization theorem can be written [2, 3, 4, 5]

$$\begin{aligned} \langle M_1 M_2 | \mathcal{Q}_i | B \rangle &= \sum_j F_J^{B \rightarrow M_1}(m_2^2) \int_0^1 du T_{ij}^I(u) \Phi_{M_2}(u) + (M_1 \rightarrow M_2) \\ &+ \int_0^1 d\xi du dv T_i^{II}(\xi, u, v) \Phi_B(\xi) \Phi_{M_1}(v) \Phi_{M_2}(u). \end{aligned} \quad (8)$$

All corrections to Eq. (8) should be suppressed by either α_s or $1/m_b$. In fact, one can perform phenomenological analysis of D and B -decays to show that amplitude behavior in the large m_Q limit is respected [6].

One can use perturbative arguments to calculate final state phases for charmless B decays using perturbative QCD [8]. Indeed, $b \rightarrow c\bar{c}s$ process, with subsequent final state rescattering of the two charmed quarks into the final state (penguin diagram) does the job, as for the energy release of the order $m_b > 2m_c$ available in b decay, the rescattered c -quarks can go on-shell generating a perturbative CP-conserving phase and thus \mathcal{A}_{CP}^{dir} , which is usually quite small for the experimentally feasible decays, $\mathcal{O}(1\%)$. One might be tempted to conclude that all strong phases in B -decays should be dominated by perturbative phases from T_{ij}^I and T_i^{II} and therefore be small. This conclusion, however, will not be correct if, for instance, real part of the decay amplitude happened to be small. Since

$$\delta \sim \alpha_s \arctan \left(\frac{Im A}{Re A} \right), \quad (9)$$

the smallness of α_s can be easily overpowered by large $1/Re A$ factor. In addition, $1/m_b$ corrections could not be computed in at this time. Yet, in this framework, it is there non-perturbative contributions to strong phases are introduced. Those contributions could be quite large.

A multitude of $B \rightarrow PP$ and $B \rightarrow PV$ transitions were evaluated in QCD factorization (QCDF) [7]. However, some predictions appear in disagreement with experimental data [9]. Among those, is the recent observation of $B \rightarrow \pi^0 \pi^0$ branching ratio, $Br(B \rightarrow \pi^0 \pi^0) = (1.5 \pm 0.5) \times 10^{-6}$, which appears to be quite larger than predicted in QCD factorization, $Br(B \rightarrow \pi^0 \pi^0) = (0.3 \pm 0.2) \times 10^{-6}$, inconsistencies in $B \rightarrow K\pi$ transitions which rule out small phases predicted by QCDF [10], as well as others [9].

A way to estimate non-perturbative contributions is to model them using approach final state interactions (FSI). In this approach, a transition $B \rightarrow M_1 M_2$ is divided into

$$\mathcal{A}(B \rightarrow M_1 M_2) = \mathcal{A}(B \rightarrow M_1 M_2) + T(M_3 M_4 \rightarrow M_1 M_2) \otimes \mathcal{A}(B \rightarrow M_1 M_2), \quad (10)$$

where M_i are the final and intermediate meson states. An important point is then how to model the rescattering amplitude $T(M_3 M_4 \rightarrow M_1 M_2)$. Most recent studies [9, 11, 12, 13] use a simple t-channel resonance exchange to model the rescattering. For example, $B \rightarrow \phi K$ transition with experimental branching ratio of $Br(B \rightarrow \phi K) = (8.6 \pm 1.1) \times 10^{-6}$ can be affected by a decay $B \rightarrow D_s^{(*)} D^{(*)}$ with a much larger branching ratios of, say, $Br(B \rightarrow D_s^* D) = (8.6 \pm 3.4) \times 10^{-3}$ and subsequent rescattering of $D_s^{(*)} D^{(*)}$ into the ϕK final state via t-channel $D_s^{(*)}$ -resonance exchange. In principle, however, other resonances, such as spin-2 D_s^{**} and others should be taken into account. A rescattering amplitude takes a very simple form,

$$T(s, t) = s^J \frac{g_{M_1 M_3} g_{M_2 M_4}}{M_J^2 - t}, \quad (11)$$

where $g_{M_i M_k}$ is a coupling constant (in practice, a t -dependent form-factor), and J is a spin of an exchanged particle. A big problem with this form of exchange amplitude is that it violates unitarity for $J \geq 2$ and thus is not appropriate for $m_b \rightarrow \infty$ limit calculations.

A solution to this problem is well-known and requires Reggeization of the scattering amplitudes [15],

$$T(s, t) = \xi \beta(t) \left(\frac{s}{s_0} \right)^{\alpha(t)} e^{i\pi\alpha(t)/2} \quad (12)$$

with $\alpha(t) = \alpha_0 + \alpha' t$ being a Regge trajectory. One consequence of this is the fact that high energy FSIs are dominated by the multiparticle intermediate states [16, 17].

$$\mathcal{A}(B \rightarrow ab) = \sum_{c,d} T(cd \rightarrow ab) \otimes \mathcal{A}(B \rightarrow cd) + \dots + \sum_{c,d,\dots,z} T(cd \dots z \rightarrow ab) \otimes \mathcal{A}(B \rightarrow cd \dots z)$$

It is known that scattering of high energy particles may be divided into ‘soft’ and ‘hard’ parts. Soft scattering occurs primarily in the forward direction with limited transverse momentum distribution which falls exponentially with a scale of order 0.5 GeV. At higher transverse momentum one encounters the region of hard scattering, which can be described by perturbative QCD. In exclusive B decay one faces the difficulty of separating the two. It might prove useful to employ unitarity in trying to describe FSI in exclusive B decays.

It is easy to investigate first the *elastic* channel. The inelastic channels have to share a similar asymptotic behavior in the heavy quark limit due to the unitarity of the \mathcal{S} -matrix. The choice of elastic channel is convenient because of the optical theorem which connects the forward (imaginary) invariant amplitude \mathcal{M} to the total cross section,

$$\text{Im } \mathcal{M}_{f \rightarrow f}(s, t=0) = 2k\sqrt{s}\sigma_{f \rightarrow \text{all}} \sim s\sigma_{f \rightarrow \text{all}} \quad , \quad (13)$$

where s and t are the usual Mandelstam variables. The asymptotic total cross sections are known experimentally to rise slowly with energy and can be parameterized by the form [14], $\sigma(s) = X(s/s_0)^{0.08} + Y(s/s_0)^{-0.56}$, where $s_0 = \mathcal{O}(1)$ GeV is a typical hadronic scale. Considering only the imaginary part of the amplitude, and building in the known exponential fall-off of the elastic cross section in t ($t < 0$) [15] by writing

$$i\text{Im } \mathcal{M}_{f \rightarrow f}(s, t) \simeq i\beta_0 \left(\frac{s}{s_0} \right)^{1.08} e^{bt} \quad , \quad (14)$$

one can calculate its contribution to the unitarity relation for a final state $f = ab$ with kinematics $p'_a + p'_b = p_a + p_b$ and $s = (p_a + p_b)^2$:

$$\begin{aligned} \text{Disc } \mathcal{M}_{B \rightarrow f} &= \frac{-i}{8\pi^2} \int \frac{d^3 p'_a}{2E'_a} \frac{d^3 p'_b}{2E'_b} \delta^{(4)}(p_B - p'_a - p'_b) \text{Im } \mathcal{M}_{f \rightarrow f}(s, t) \mathcal{M}_{B \rightarrow f} \\ &= -\frac{1}{16\pi} \frac{i\beta_0}{s_0 b} \left(\frac{m_B^2}{s_0} \right)^{0.08} \mathcal{M}_{B \rightarrow f} \quad , \end{aligned} \quad (15)$$

where $t = (p_a - p'_a)^2 \simeq -s(1 - \cos \theta)/2$, and $s = m_B^2$.

One can refine the argument further, since the phenomenology of high energy scattering is well accounted for by the Regge theory [15]. In the Regge model, scattering amplitudes are described by the exchanges of Regge trajectories (families of particles of differing spin) with the leading contribution given by the Pomeron exchange. Calculating the Pomeron contribution to the elastic final state rescattering in $B \rightarrow \pi\pi$ one finds [16]

$$\text{Disc } \mathcal{M}_{B \rightarrow \pi\pi} |_{\text{Pomeron}} = -i\epsilon \mathcal{M}_{B \rightarrow \pi\pi}, \quad \epsilon \simeq 0.21 \quad . \quad (16)$$

It is important that the Pomeron-exchange amplitude is seen to be almost purely imaginary. However, of chief significance is the identified weak dependence of ϵ on m_B – the $(m_B^2)^{0.08}$ factor in the numerator is attenuated by the $\ln(m_B^2/s_0)$ dependence in the effective value of b .

The analysis of the elastic channel suggests that, at high energies, FSI phases are *mainly generated by inelastic effects*, which follows from the fact that the high energy cross section is mostly inelastic. This also follows from the fact that the Pomeron elastic amplitude is almost purely imaginary. Since the study of elastic rescattering has yielded a \mathcal{T} -matrix element $\mathcal{T}_{ab \rightarrow ab} = 2i\epsilon$, i.e. $\mathcal{S}_{ab \rightarrow ab} = 1 - 2\epsilon$, and since the constraint of unitarity of the \mathcal{S} -matrix implies that the off-diagonal elements are $\mathcal{O}(\sqrt{\epsilon})$, with ϵ approximately $\mathcal{O}(m_B^0)$ in powers of m_B and numerically $\epsilon < 1$, then the inelastic amplitude must also be $\mathcal{O}(m_B^0)$ with $\sqrt{\epsilon} > \epsilon$. Similar conclusions follow from the consideration of the final state unitarity relations. This complements the old Bjorken picture of heavy meson decay (the dominance of the matrix element by the formation of the small hadronic configuration which grows into the final state pion “far away” from the point it was produced and does not interact with the soft gluon fields present in the decay, see also [21] for the discussion) by allowing for the rescattering of multiparticle states, production of whose is favorable in the $m_b \rightarrow \infty$ limit, into the two body final state. Analysis of the final-state unitarity relations in their general form is complicated due to the many contributing intermediate states, but we can illustrate the systematics of inelastic scattering in a two-channel model. It involves a two-body final state f_1 undergoing elastic scattering and a final state f_2 which represents ‘everything else’. As before, the elastic amplitude is purely *imaginary*, which dictates the following *one-parameter* form for the S matrix

$$S = \begin{pmatrix} \cos 2\theta & i \sin 2\theta \\ i \sin 2\theta & \cos 2\theta \end{pmatrix}, \quad T = \begin{pmatrix} 2i \sin^2 \theta & \sin 2\theta \\ \sin 2\theta & 2i \sin^2 \theta \end{pmatrix}, \quad (17)$$

where we identify $\sin^2 \theta \equiv \epsilon$. The unitarity relations become

$$\begin{aligned} \text{Disc } \mathcal{M}_{B \rightarrow f_1} &= -i \sin^2 \theta \mathcal{M}_{B \rightarrow f_1} + \frac{1}{2} \sin 2\theta \mathcal{M}_{B \rightarrow f_2}, \\ \text{Disc } \mathcal{M}_{B \rightarrow f_2} &= \frac{1}{2} \sin 2\theta \mathcal{M}_{B \rightarrow f_1} - i \sin^2 \theta \mathcal{M}_{B \rightarrow f_2} \end{aligned} \quad (18)$$

Denoting \mathcal{M}_1^0 and \mathcal{M}_2^0 to be the decay amplitudes in the limit $\theta \rightarrow 0$, an exact solution to Eq. (18) is given by

$$\mathcal{M}_{B \rightarrow f_1} = \cos \theta \mathcal{M}_1^0 + i \sin \theta \mathcal{M}_2^0, \quad \mathcal{M}_{B \rightarrow f_2} = \cos \theta \mathcal{M}_2^0 + i \sin \theta \mathcal{M}_1^0. \quad (19)$$

Thus, the phase is given by the inelastic scattering with a result of order

$$\text{Im } \mathcal{M}_{B \rightarrow f} / \text{Re } \mathcal{M}_{B \rightarrow f} \sim \sqrt{\epsilon} \left(\mathcal{M}_2^0 / \mathcal{M}_1^0 \right). \quad (20)$$

Clearly, for physical B decay, we no longer have a simple one-parameter \mathcal{S} matrix, and, with many channels, cancellations or enhancements are possible for the sum of many contributions. However, the main feature of the above result is expected to remain: inelastic channels cannot vanish and provide the non-perturbative FSI phase. Further studies of flavor-nondiagonal transitions showed that FSIs are suppressed by powers of $1/m_b$ [18, 19, 20] and thus can be used to estimate nonperturbative strong phases in the heavy quark limit.

3. Conclusions

I reviewed the physics of final state interactions in meson decays. I showed that the use of pole exchange graphs to model FSI amplitudes leads to violation of unitarity in the large m_b limit,

so consistent studies of high-energy rescattering amplitudes must involve Reggiazed rescattering amplitudes. Several examples were provided.

One of the main goals of physics of CP violation and meson decay is to correctly extract the underlying parameters of the fundamental Lagrangian that are responsible for these phenomena. The understanding of final state interactions is very important for the success of this program.

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